# Notes on ultrasonic phase and group velocity measurements.

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This is a topic that has been confusing to many of our users. It is, therefore, the purpose of this note to explain some of the difficulties in measuring absolute phase velocity.

Instruments manufactured by Ritec mainly use radio frequency bursts of energy, which are derived from an accurate frequency synthesizer. However, a burst is not monochromatic in nature but must contain a spectrum of frequencies centered about the carrier (synthesizer) frequency. The width of this spectrum is an inverse function of the pulse width and therefore becomes wider as the pulse width is narrowed.

If the wave speeds of all the frequencies in this spectrum are the same (non-dispersive), the group velocity of this burst and the phase velocity of the cycles within the burst will be equal. If the wave speed of the higher frequencies in the spectrum is lower than the speed of the lower frequencies (dispersive), the group will proceed at a slower rate than the individual cycles. The phase and group velocities will not be equal.

In order to help visualize this phenomenon in a simple way, we will consider only two frequencies $f_1$ and $f_2$ ($\omega_1$ and $\omega_2$) will be considered. Two one dimensional waves added together give

$$y = A\cos(k_1 x - w_1 t) + \\ + A\cos(k_2 x - w_2 t) \qquad (1)$$

where the phase velocities of the frequencies are given by

$$\frac{w_1}{k_1} = c_1, \quad \frac{w_2}{k_2} = c_2 . \qquad (2)$$

When we substitute $k_1 = 2\pi/\lambda_1$ and $\omega_1 = 2\pi f$, we have the familiar expression

$$\lambda_1 f_1 = c_1 \qquad (3)$$

Equation 1 can be rewritten using the trigonometric identity

$$2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = \\ = \cos B + \cos C \qquad (4)$$

The result is

$$y = 2A\cos\left[\left(\frac{k_2 - k_1}{2}\right)x - \left(\frac{w_2 - w_1}{2}\right)t\right]\times \\ \times\cos\left[\left(\frac{k_2 + k_1}{2}\right)x - \left(\frac{w_2 + w_1}{2}\right)t\right] \qquad (5)$$

We define the average wave number and frequency as

$$k = \frac{k_2 + k_1}{2} \quad , \quad w = \frac{w_2 + w_1}{2} \qquad (6)$$

and the difference terms as

$$\Delta k = k_2 - k_1 \quad , \quad \Delta w = w_2 - w_1 \qquad (7)$$

Equation 5 may now be rewritten as

$$y = 2A\cos\left(\frac{\Delta k x}{2} - \frac{\Delta w t}{2}\right)\times\cos(kx - wt) \qquad (8)$$

where the first cos term can be thought of as a low frequency multiplicative modulation and the second term as the high frequency carrier. This can be viewed as a series of groups all traveling at the same group speed.

The phase velocity of the carrier is

$$c_\phi = \omega/k \qquad (9)$$

and the velocity of the low frequency modulation is

$$c_g = \Delta\omega/\Delta k. \qquad (10)$$

The following plot of $\omega$ versus k for a hypothetical spectrum illustrates the situation. At point (a) in Fig.1 the phase velocity is represented by the slope of the line drawn from that point to the origin, whereas the group velocity is represented by the slope of the tangent to the curve at that point. In this case $c_g < c_\phi$ . Point (b) represents the case where $c_g > c_\phi$ .

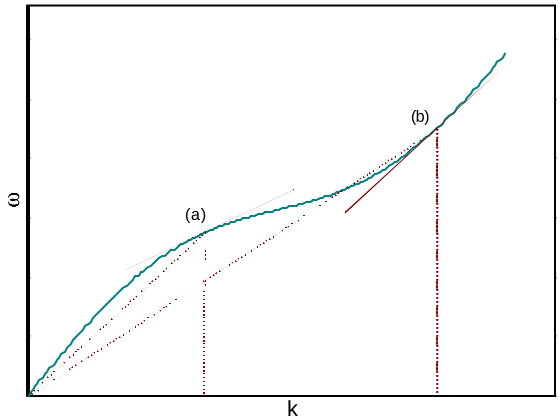


Fig. 1
Hypothetical spectrum illustrating different phase and group velocities

The situation is shown in the following three figures, which show both the carrier and the modulation waveforms. Five pictures of the wave are shown at one-microsecond intervals and the central carrier cycle of the group (t=0) has been marked so that its position can be followed as the wave proceeds. Only one group is shown after the first in order to simplify the picture.

Note: In all the examples f is arbitrarily chosen as 1 MHz ($\omega = 6.28*10^6$ rad/sec). This was done by choosing initially $f_1 = 0.9$ MHz and $f_2 = 1.1$ MHz. The ks were calculated using $c = 5*10^5$ cm/sec and remain constant for all the simulations. The frequencies were adjusted in Figures 3 & 4 to satisfy the relations $\omega_1 = k_1 c_1$, etc. The average frequency (f) is always 1 MHz because the chosen deviations in velocity average to zero.

Fig. 2 shows the situation for the case where the group and phase velocity are equal.

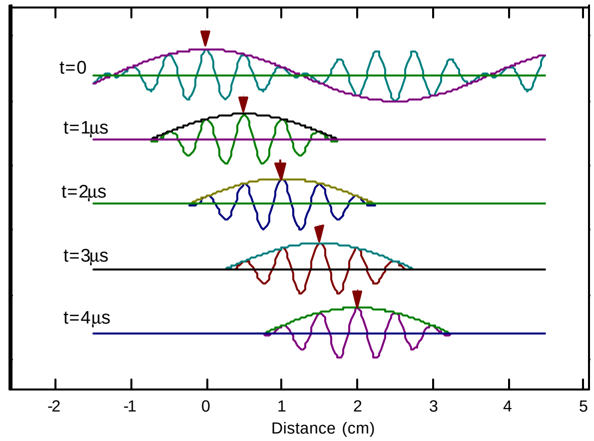


Fig. 2
$f = 1$ MHz, $\omega = 6.28*10^6$ rad/sec
$k_1 = 11.31$ rad/cm, $k_2 = 13.823$ rad/cm
$\omega_1 = 5.655*10^6$ rad/sec, $\omega_2 = 6.91*10^6$ rad/sec
$c_1 = 5*10^5$ cm/sec, $c_2 = 5*10^5$ cm/sec
$c_g = c_\phi = 5*10^5$ cm/sec

Fig. 3 examines the case where $c_1 < c_2$ . The wave numbers ($k_1$, $k_2$) remain the same but the frequencies have been adjusted so that
$\omega_1 = k_1 c_1$, etc.

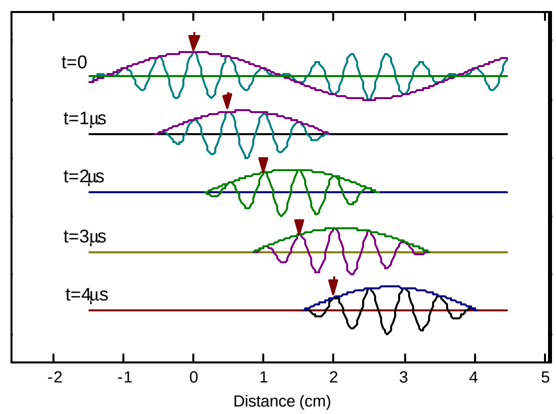


Fig. 3
$f = 1$ MHz, $\omega = 6.28*10^6$ rad/sec
$k_1 = 11.31$ rad/cm, $k_2 = 13.823$ rad/cm
$\omega_1 = 5.43*10^6$ rad/sec, $\omega_2 = 7.19*10^6$ rad/sec
$c_1 = 4.8*10^5$ cm/sec, $c_2 = 5.2*10^5$ cm/sec
$c_g = 7*10^5$ cm/sec, $c_\phi = 5*10^5$ cm/sec

As can be seen, the modulation waveform moves faster than the carrier so that at t = 4µs, the marked cycle is no longer at the center of the wavepacket.

Fig. 4 examines the case where $c_1 > c_2$ . The wave numbers ($k_1$, $k_2$) remain the same, but the frequencies have been adjusted so that
$\omega_1 = k_1 c_1$, etc.

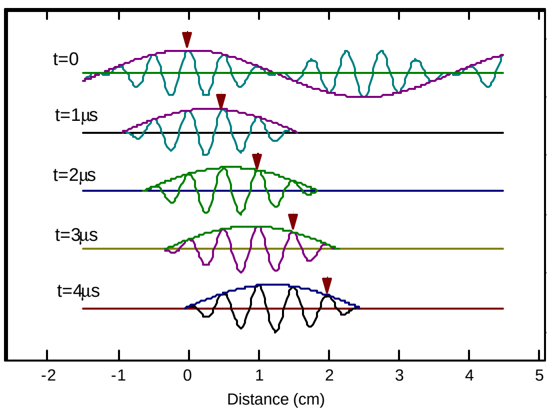


Fig. 4

f = 1 MHz, $\omega = 6.28*10^6$ rad/sec

$k_1 = 11.31$ rad/cm, $k_2 = 13.823$ rad/cm

$\omega_1 = 5.88*10^6$ rad/sec, $\omega_2 = 6.635*10^6$ rad/sec

$c_1 = 5.2*10^5$ cm/sec, $c_2 = 4.8*10^5$ cm/sec

$c_g = 3*10^5$ cm/sec, $c_\phi = 5*10^5$ cm/sec

As can be seen, the modulation waveform moves slower than the carrier and the marked cycle moves toward the end of the group.

Equation 9 can't be used to make bulk phase velocity measurements in solids with Ritec instrumentation because k is not known. The situation with liquids or surface waves is much easier. The receive transducer can be moved; the phase angle monitored; and the wavelength determined. This gives an unambiguous result for the phase velocity and involves no corrections for frequency dependant factors.

However, some progress, even in solids, can be made by measuring the phase of the received signal. It is possible for us to determine the group transit time and very accurate measurements of the change in phase transit time can be made. Consider equation 9 again.

$$c_f = \frac{w}{k} = \frac{x}{t_f} \tag{11}$$

After solving for $t_\phi$ equation 11 becomes

$$t_f = \frac{kx}{w} = \frac{f}{2pf} \tag{12}$$

where $\phi$ is an absolute phase angle (usually many times 2 $\pi$). Unfortunately, we only have the ability of measuring the phase difference between the carrier and the received signal and this is limited to angles less than 2 $\pi$. We can, however, track changes in the transit time as a function of time, temperature, or some other parameter.

$$\Delta t_f = \frac{\Delta f}{2pf} \tag{13}$$

Equation 10 was derived using only two frequencies. A similar expression can be obtained using the frequency spectrum of a single RF burst and the results are

$$c_g = \frac{dw}{dk} \tag{14}$$

which gives

$$t_g = \frac{1}{2p}\frac{df}{df} \tag{15}$$

It is possible for us to measure the slope of the phase frequency curve and therefore obtain the group transit time. However, the researcher is cautioned that it is absolutely necessary to apply corrections for the transducer, the bond, diffraction, and any frequency sensitive part of the experimental apparatus in order to obtain an accurate result. A hypothetical situation is shown in Fig. 5. In this case the point (a) indicates a larger $t_g$ than $t_\phi$ because the slope of the tangent is greater than the slope of the line from (a) to the origin.

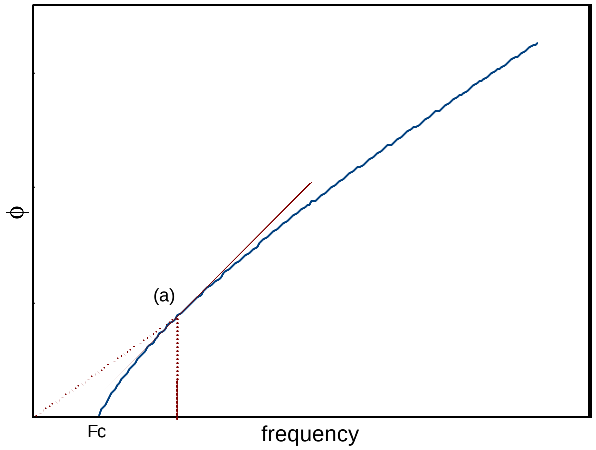


Fig. 5
Hypothetical phase versus frequency plot

Note: We normally can't measure $t_\phi$ because usually we have no information about the phase in the low frequency range. We are only able to make measurements within the bandwidth of the transducer. At very low frequencies the size of the sample will also present problems. Ultrasonic frequency measurements can, however, indicate the presence of dispersion if the phase versus frequency curve deviates significantly from a straight line.

It should also be mentioned that the simple treatment of this subject in this note uses one-dimensional waves. When the situation involves three dimensions, care must be made to include the effects of diffraction and any other frequency dependent factors.

There is one way that the absolute phase velocity can be made in solid samples with certain defined geometrical shapes. Long RF burst widths can be used to determine a series of resonant frequencies. In a flat plate the condition for resonance is very simple. The thickness (h) must contain an integer number of half wavelengths.

$$N\lambda = 2h \qquad (16)$$

which gives after substituting $\lambda$ from equation 3 and using successive resonances

$$c_\phi = 2h\Delta f \qquad (17)$$

where $\Delta f$ is the frequency difference between resonances.

This method works well when the transducer can be partially decoupled from the sample. It should not be a part of the resonant system.